Modeling the Cardiovascular System of the Human Body

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ABSTRACT

This paper discusses how sensitivity analysis can be used to model the human cardiovascular system. Our goal is to better understand the relevance of each section and subsection of arteries, veins, and capillaries. We model the pressure, flow, and volume through different parts of the system over time and examine how changing different parameters (resistance, compliance, elastance, and timing) changes the model. We also examine the sensitivity of the model with respect to these parameters, and model the sensitivities. We conducted our research with reference to common predetermined diagrammatic cardiovascular models. The findings of this paper are intended to better the understanding of blood flow throughout specific paths in the body.

INTRODUCTION

Modeling the cardiovascular system is necessary to make improvements in medical fields. With a better model of the system, we have a better understanding of how the individual parts interact to make up the complete system. This model could potentially then be utilized in medicine to help understand, cure, and prevent cardiovascular diseases. In this paper, we explore the way blood circulates through veins, arteries, and capillaries, and how its behavior changes given different parameters. We examine and discuss the change in resistance, volume, pressure, and flow of blood as it travels throughout the system.

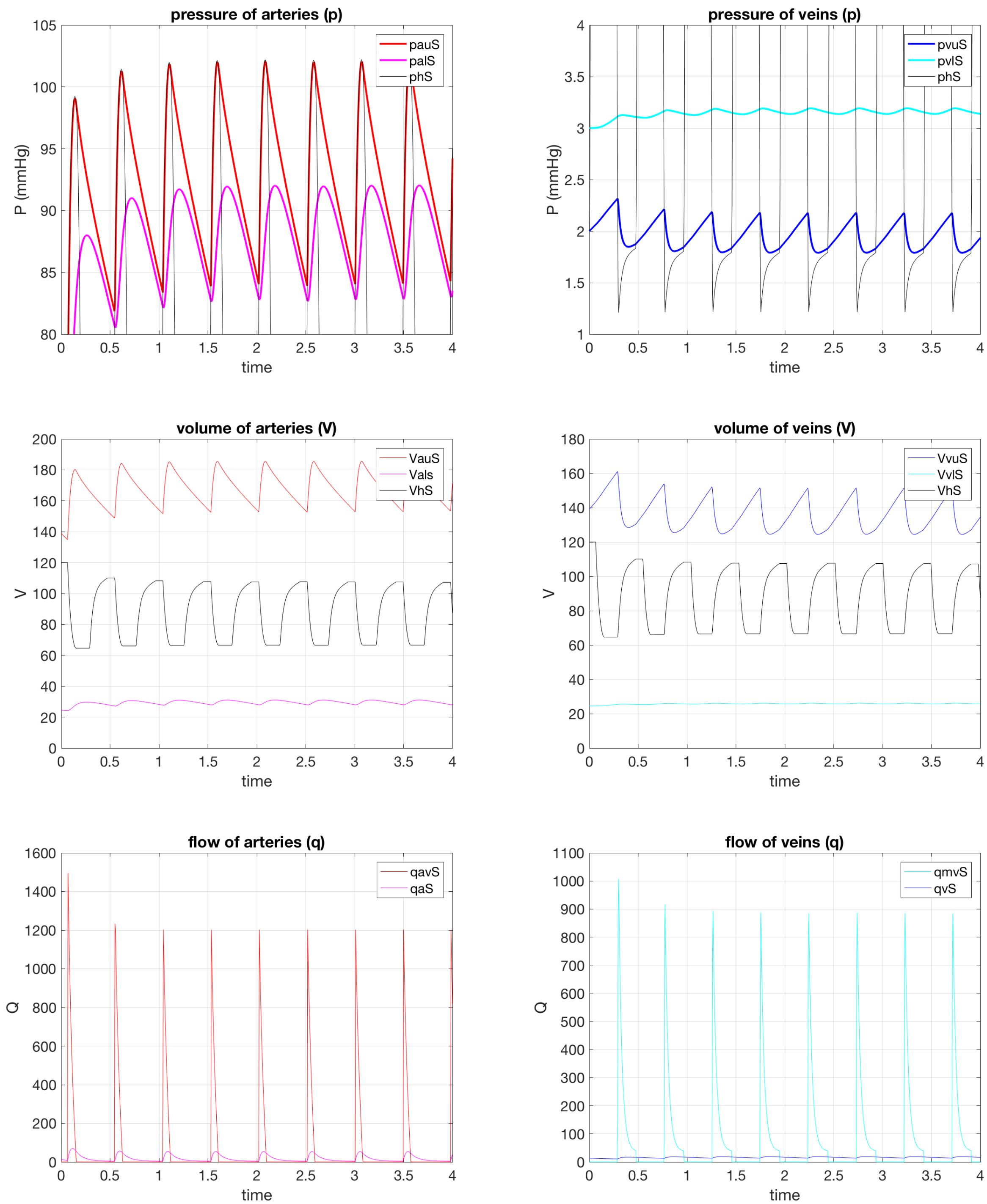
METHODS

To conduct our research, we collected data from ten volunteers aged 20-23 years (with the average age 21.2). To collect heart data, we used a blood pressure gauge (placed on their left arm) and a noninvasive heart rate monitor (on the finger of the same arm). Data was taken from each volunteer from an initial sitting position, then the volunteers were instructed to stand. This allows us to study the cardiovascular system in a stressed, and unstressed condition. To negate the effects of a gravitational pull on the system, each volunteer's left arm was strapped to their chest such that it rested at heart level. Data was collected for several seconds, under each stressed and unstressed conditions.

Initially, the pressures, flows, and volumes at key points throughout the cycle were modeled with respect to time for individual students in the study. Following this, different parameters were modified one at a time to examine the effect they have on the overall system. The resistance, compliance, elastance, and timing of the system were changed and the resulting pressures, flows, and volumes were modeled again.

To quantitatively measure the sensitivity of the different parameters on the model, a computational sensitivity analysis was performed (see Results 2C). Sensitivity equations were derived analytically for several parameters as well. The parameters which were found to not be correlated were varied together to observe the effect on the model.

RESULTS



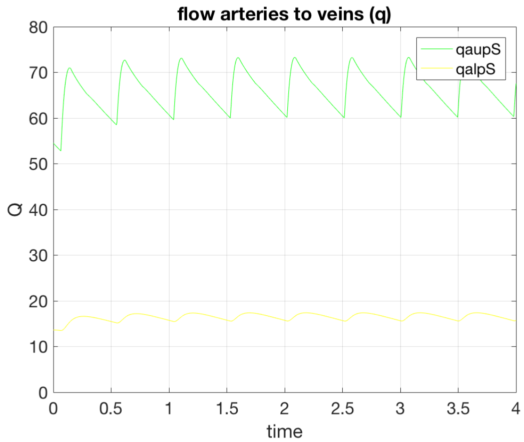
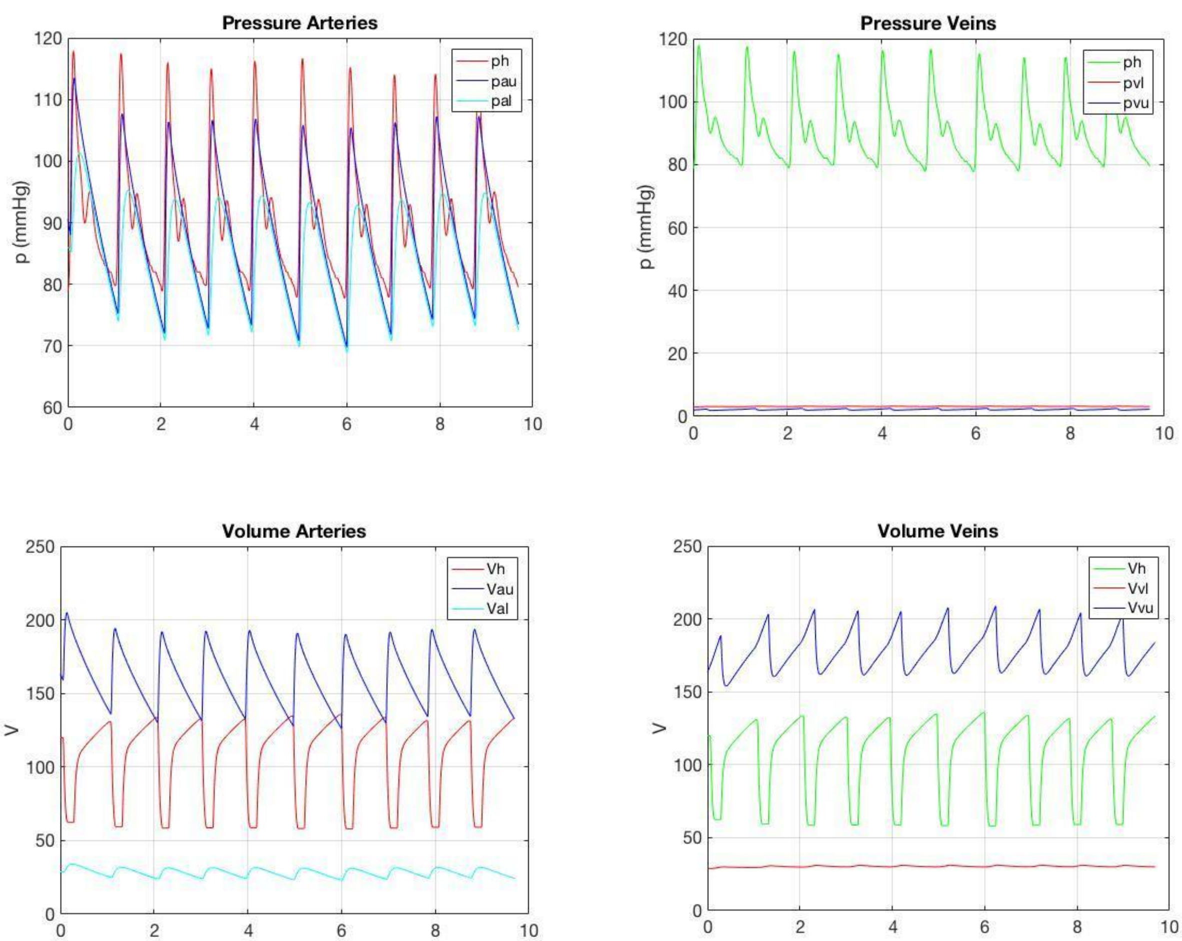


Figure 1. Data for Student 1.



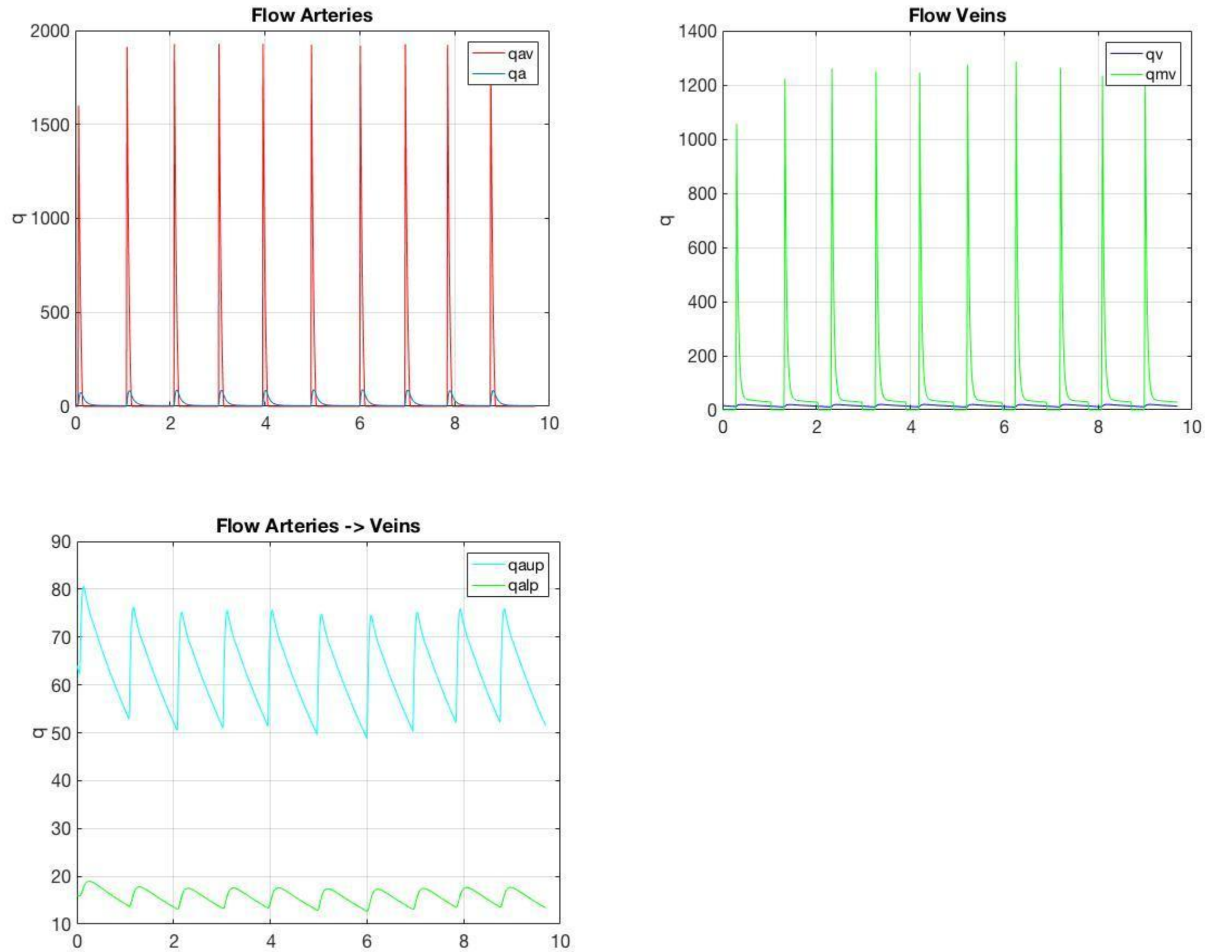


Figure 2. Data for Student 11.

The preceding graphs depict the pressure, volume, and flow of arteries and veins, as well as the flow between the two. The data points portrayed in these graphs are from the original lab tests we previously conducted. As you can see, there are natural differences amongst the students due to differences in genetics, activity level, diet, and other possible lifestyle choices. However, it is clear that, generally, the plots are similar. You can notice an oscillation in each parameter as time continues, varying in degree proportionally to the heart's beat.

Parameter Analysis

We halved each parameter (R_{aup}, R_{aup},C_{au}, C_{al}, C_{vu}, C_{vl}, EM,Em) and studied the resulting effects on Pressure (P_{au},P_{al}, P_{vu}, P_{vl}) and Volume (V_{au},V_{al}, V_{vu}, V_{vl}) . The findings are as follows:

Halving R_{aup}

(i) Shifted down P_{au} and P_{al} each by ~10 mmHg

(ii) V_{au} decreases maxima by ~20 L, decreases minima by ~40 L

(iii) V_{vu} increases maxima by ~40 L, decreases minima by ~20 L

(iv) V_{al} shifts down slightly

Halving R_{aup}

(i) Shifted down P_{au} and P_{al} each by ~10 mmHg

(ii) V_{vu}, V_{au}, V_{vl} increase by ~5 L

Halving C_{au}

(i) Increased oscillations of P_{au} and P_{al}(~60 mmHg → 140 mmHg)

(ii) V_{vu} shifted up by ~50 L

(iii) V_{au} shifted down by ~100 L

(iv) V_{al} begins higher (~50 L) then decreases to slightly above normal levels with higher amplitude.

(v) V_{vl} is slightly higher

Halving C_{al}

(i) Decrease the oscillation minima of P_{au} and P_{al}

(ii) Increased the difference between P_{au} and P_{al}

(iii) V_{al} shifted down by ~10 L

Halving C_{vu}

(i) Increased P_{au} and P_{al} oscillations by ~10 mmHg

(ii) Increased the difference between P_{au} and P_{al}

(iii) Increased amplitude of P_{vu}

(iv) Increased amplitude of P_{vl} by ~1 mmHg

(v) V_{au} increased maxima by ~50 L, minima by ~30 L.

(vi) V_{vu} shifted down by ~60 L

(vii) V_{al} shited up slightly

Halving C_{vl}

(i) P_{vl} starts high ~6 mmHg then decreases to ~4 mmHg (which is ~1 mmHg higher than normal)

(ii) V_{vl} levels decline initially until halved then becomes steady

Halving EM

(i) Slightly lowers amplitude of P_{au} and P_{al} oscillations

(ii) V_{au} shifted down by ~40 L, decreasing amplitude

(iii) V_{vu} shifted up by ~20 L

(iv) V_{al} declines initially then levels out ~5 L below normal levels

Halving Em

(i) Increase amplitude of P_{au} and P_{al} oscillations by ~10 mmHg

(ii) P_{vu} decreased maxima and minima by ~1 mmHg

(iii) V_{au} increases amplitude and shifts up by ~50 L

Sensitivity Analysis

Analytical

We used a rudimentary set of ordinary differential equations to model the system. We derived the sensitivity equations for the volume of the lower body arteries $V_{al}(t)$ with respect to the lower body peripheral resistance $R_{alp}(t)$, minimum elastance $Em(t)$, and compliance of lower body arteries $C_{al}(t)$. This system of equations is derived as follows,

$$\frac{\alpha}{\alpha t}\left(\frac{\alpha V_{al}}{\alpha R_{alp}}\right) = \frac{\alpha}{\alpha R_{alp}}\frac{\alpha V_{al}}{\alpha t}$$

$$=\frac{\alpha}{\alpha R_{alp}}\left(\frac{V_{au}}{R_a C_{au}} - \frac{V_{al}}{R_a C_{al}} - \frac{V_{al}}{R_{alp} C_{al}} + \frac{V_{vl}}{R_{alp} C_{vl}}\right)$$

$$= \frac{\frac{\alpha V_{au}}{\alpha R_{alp}}}{R_a C_{au}} - \frac{\frac{\alpha V_{al}}{\alpha R_{alp}}}{R_a C_{al}} - \frac{\frac{\alpha V_{al}}{\alpha R_{alp}}}{R_{alp} C_{al}} + \frac{\frac{\alpha V_{al}}{\alpha R_{alp}}}{R_{alp} C_{vl}} + \frac{V_{al}}{R^2_{alp} C_{al}} - \frac{V_{vl}}{R^2_{alp} C_{vl}}$$

$$\frac{\alpha}{\alpha t}\left(\frac{\alpha V_{al}}{\alpha E_m}\right) = \frac{\alpha}{\alpha E_m}\frac{\alpha V_{al}}{\alpha t}$$

$$= \frac{\alpha}{\alpha E_m}\left(\frac{V_{au}}{R_a C_{au}} - \frac{V_{al}}{R_a C_{al}} - \frac{V_{al}}{R_{alp} C_{al}} + \frac{V_{vl}}{R_{alp} C_{vl}}\right)$$

$$= \frac{\frac{\alpha V_{au}}{\alpha E_m}}{R_a C_{au}} - \frac{\frac{\alpha V_{al}}{\alpha E_m}}{R_a C_{al}} - \frac{\frac{\alpha V_{al}}{\alpha E_m}}{R_{alp} C_{al}} + \frac{\frac{\alpha V_{al}}{\alpha E_m}}{R_{alp} C_{vl}}$$

$$\frac{\alpha}{\alpha t}\left(\frac{\alpha V_{al}}{\alpha C_{al}}\right) = \frac{\alpha}{\alpha C_{al}}\frac{\alpha V_{al}}{\alpha t}$$

$$= \frac{\alpha}{\alpha C_{al}}\left(\frac{V_{au}}{R_a C_{au}} - \frac{V_{al}}{R_a C_{al}} - \frac{V_{al}}{R_{alp} C_{al}} + \frac{V_{vl}}{R_{alp} C_{vl}}\right)$$

$$= \frac{\frac{\alpha V_{au}}{\alpha C_{al}}}{R_a C_{au}} - \frac{\frac{\alpha V_{al}}{\alpha C_{al}}}{R_a C_{al}} - \frac{\frac{\alpha V_{al}}{\alpha C_{al}}}{R_{alp} C_{al}} + \frac{\frac{\alpha V_{al}}{\alpha C_{al}}}{R_{alp} C_{vl}} + \frac{V_{al}}{C^2_{al} R_a} + \frac{V_{al}}{C^2_{al} R_{alp}}$$

Computational

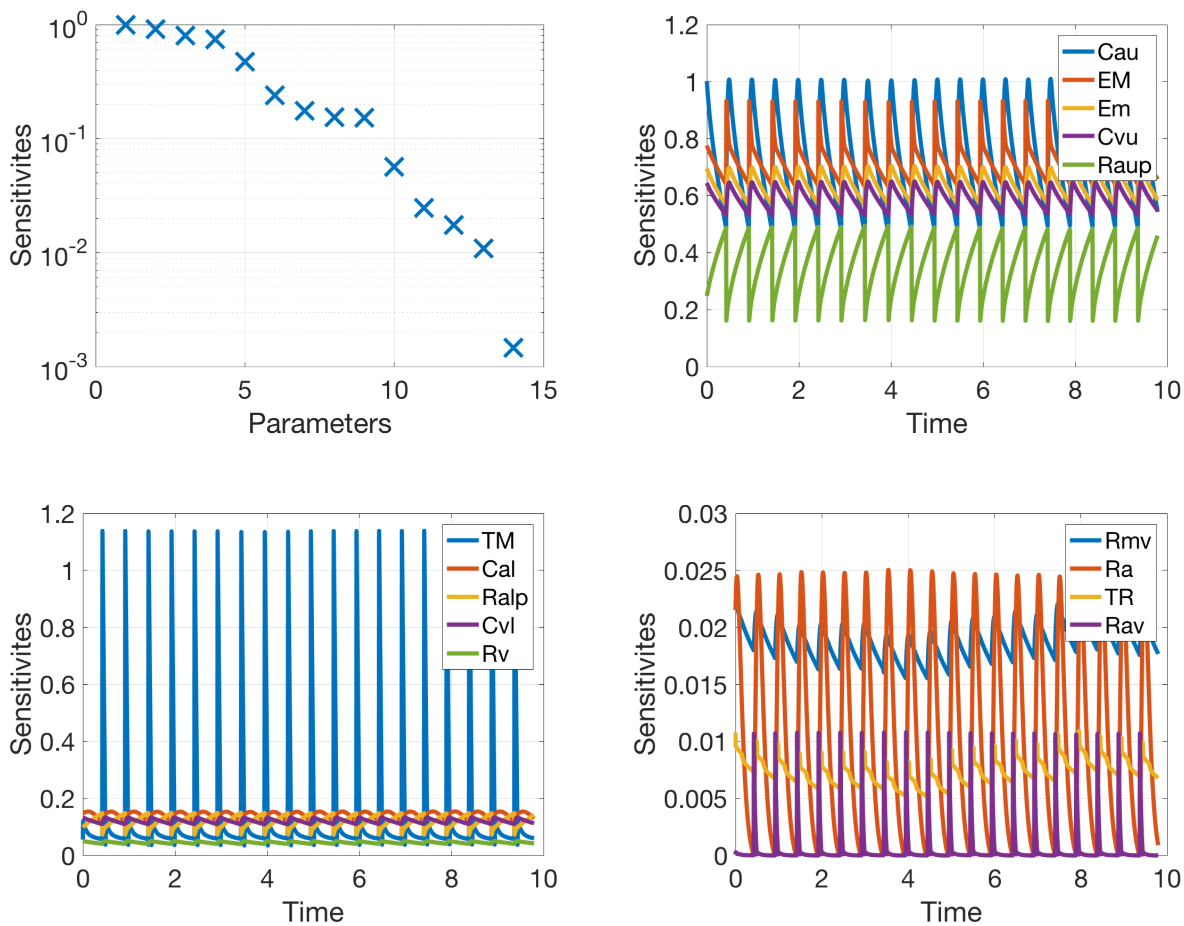


Figure 3. Sensitivity analysis for Student 1.

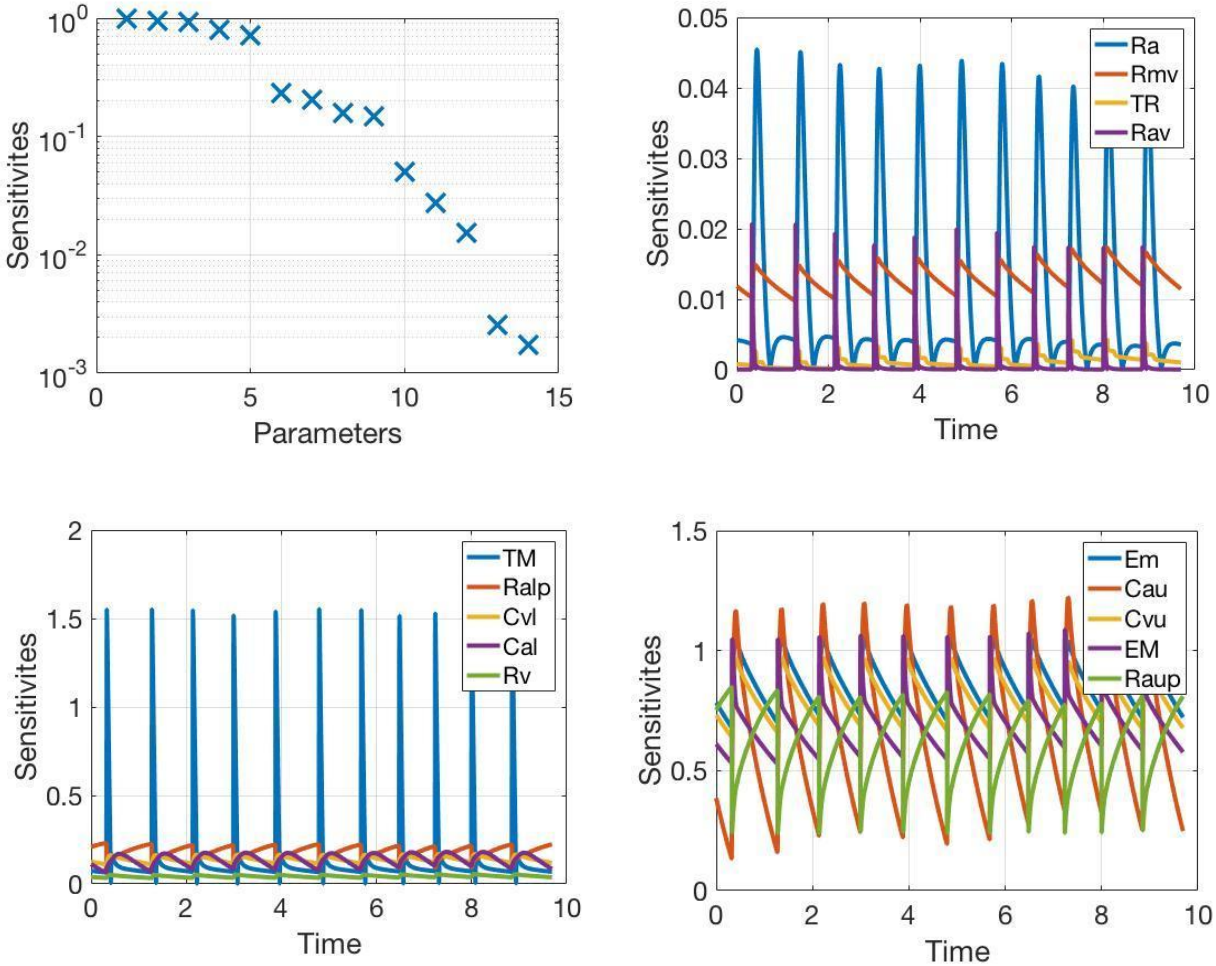


Figure 4. Sensitivity analysis for Student 11.

Covariance Matrix: Student 1				
0	-0.6	0	0	-0.6
0	0	-0.4	0.1	0.5
0	0	0	-0.1	-0.1
0	0	0	0	-0.8
0	0	0	0	0

Covariance Matrix: Student 11					
0	-0.7	0.9	-0.3	0.03	0
0	0	-0.5	0.1	0.5	-0.3
0	0	0	-0.3	0.3	-0.1
0	0	0	0	-0.1	0.02
0	0	0	0	0	-0.4
0	0	0	0	0	0

Figure 5. Covariance matrices of Student 1 and Student 11.

Student 1

Parameter	S1 = [7 1 13 3 6]	S1 = [7 1 13]	% Error
Raup	**0.611**	**0.5576**	**8.73977087**
Ra	0.2971	0.2971	0
Rv	**0.0397**	**0.0777**	**95.7178841**
Ralp	5.4123	5.4123	0
Rav	0.001	0.001	0
Rmv	**0.0039**	**0.001**	**74.3589744**
Cau	**1.4437**	**1.5849**	**9.7804253**
Cal	0.3188	0.3188	0
Cvu	65.6184	65.6184	0
Cvl	7.7198	7.7198	0
EM	1.5292	1.5292	0
Em	0.0167	0.0167	0
TM	**0.0715**	**0.0724**	**1.25874126**
TR	0.15	0.15	0

Student 11

S1 = [12 7 1 13 2 14]	S1 = [12 7 1]	Error
0.4196	0.3414	18.6367969
0.2824	0.2804	0.7082153
0.0734	0.0734	0
5.1084	5.1084	0
0.001	0.001	0
0.001	0.001	0
2.048	2.048	0
0.3378	0.3378	0
69.5217	69.5217	0
8.179	8.179	0
1.5292	1.5292	0
0.0081	0.0057	29.6296296
0.0718	0.15	108.913649
0.3577	0.15	58.0654179

Figure 6. Estimating the parameter subset for Student 1 and Student 11.

Student 1

Parameter	Unscaled	Scaled	% Error
Raup	**0.5518**	**0.611**	**10.7285248**
Ra	0.2971	0.2971	0
Rv	**0.0718**	**0.0397**	**44.7075209**
Ralp	5.4123	5.4123	0
Rav	0.001	0.001	0
Rmv	**0.001**	**0.0039**	290
Cau	**1.6049**	**1.4437**	**10.0442395**
Cal	0.3188	0.3188	0
Cvu	65.6184	65.6184	0
Cvl	7.7198	7.7198	0
EM	1.5292	1.5292	0
Em	0.0167	0.0167	0
TM	**0.0702**	**0.0715**	**1.85185185**
TR	0.15	0.15	0
Gradient	0.0007	1.3995	
Cost	0.0073	41.862	

Student 11

	Unscaled	Scaled	Error
Raup	0.69	0.67	3.99
Raup	0.69	0.47	31.89
Rv	0.06	0.06	0.00
Ralp	5.80	5.80	0.00
Rav	0.00	0.00	0.00
Rmv	0.00	0.00	0.00
Cau	1.98	1.97	0.90
Cal	0.30	0.30	0.00
Cvu	81.69	81.69	0.00
Cvl	9.61	9.61	0.00
EM	2.02	2.02	0.00
Em	0.01	0.00	26.32
TM	0.09	0.10	7.62
TR	0.23	0.59	154.95
Gradient	0.00	1.6	
Cost	0.01	81.3	

Figure 7. Comparing scaled and unscaled optimization costs and gradients for Student 1 and Student 11.

Student 1

	Original	0.01	0.1	0.9	1.1	11
Raup	**0.611**	**0.6324**	**0.5804**	**0.5709**	**0.544**	**0.5326**
Ra	0.2971	0.2971	0.2971	0.2971	0.2971	0.2971
Rv	**0.0397**	**0.1244**	**0.0194**	**0.0964**	**0.059**	**0.0206**
Ralp	5.4123	5.4123	5.4123	5.4123	5.4123	5.4123
Rav	0.001	0.001	0.001	0.001	0.001	0.001
Rmv	**0.0039**	**0.0003**	**0.0003**	**0.0022**	**0.0005**	**0.0003**
Cau	**1.4437**	**1.0403**	**1.1999**	**1.5154**	**1.6266**	**1.6599**
Cal	0.3188	0.3188	0.3188	0.3188	0.3188	0.3188
Cvu	65.6184	65.6184	65.6184	65.6184	65.6184	65.6184
Cvl	7.7198	7.7198	7.7198	7.7198	7.7198	7.7198
EM	1.5292	1.5292	1.5292	1.5292	1.5292	1.5292
Em	0.0167	0.0167	0.0167	0.0167	0.0167	0.0167
TM	**0.0715**	**0.6**	**0.3523**	**0.083**	**0.0705**	**0.0708**
TR	0.15	0.15	0.15	0.15	0.15	0.15

Student 11

	Original	X = 0.9	X = 1.1	X = 0.1	X= 0.01	X = 11
Raup	0.4196	0.6874	0.6497	0.675	0.6966	0.6366
Raup	0.2824	1.0712	1.0669	0.7654	0.9432	0.3859
Rv	0.0734	0.0624	0.0624	0.0624	0.0624	0.0624
Ralp	5.1084	5.8011	5.8011	5.8011	5.8011	5.8011
Rav	0.001	0.001	0.001	0.001	0.001	0.001
Rmv	0.001	0.001	0.001	0.001	0.001	0.001
Cau	2.048	1.9537	2.1106	2.0097	1.9347	2.082
Cal	0.3378	0.3006	0.3006	0.3006	0.3006	0.3006
Cvu	69.5217	81.6877	81.6877	81.6877	81.6877	81.6877
Cvl	8.179	9.6103	9.6103	9.6103	9.6103	9.6103
EM	1.5292	2.0193	2.0193	2.0193	2.0193	2.0193
Em	0.0081	0.0047	0.0044	0.0051	0.0049	0.0042
TM	0.0718	0.0977	0.0969	0.1017	0.0943	0.1039
TR	0.3577	0.5939	0.0415	0.1365	0.5992	0.0427

Figure 8. Table showing the optimized parameter given various nominal parameters.

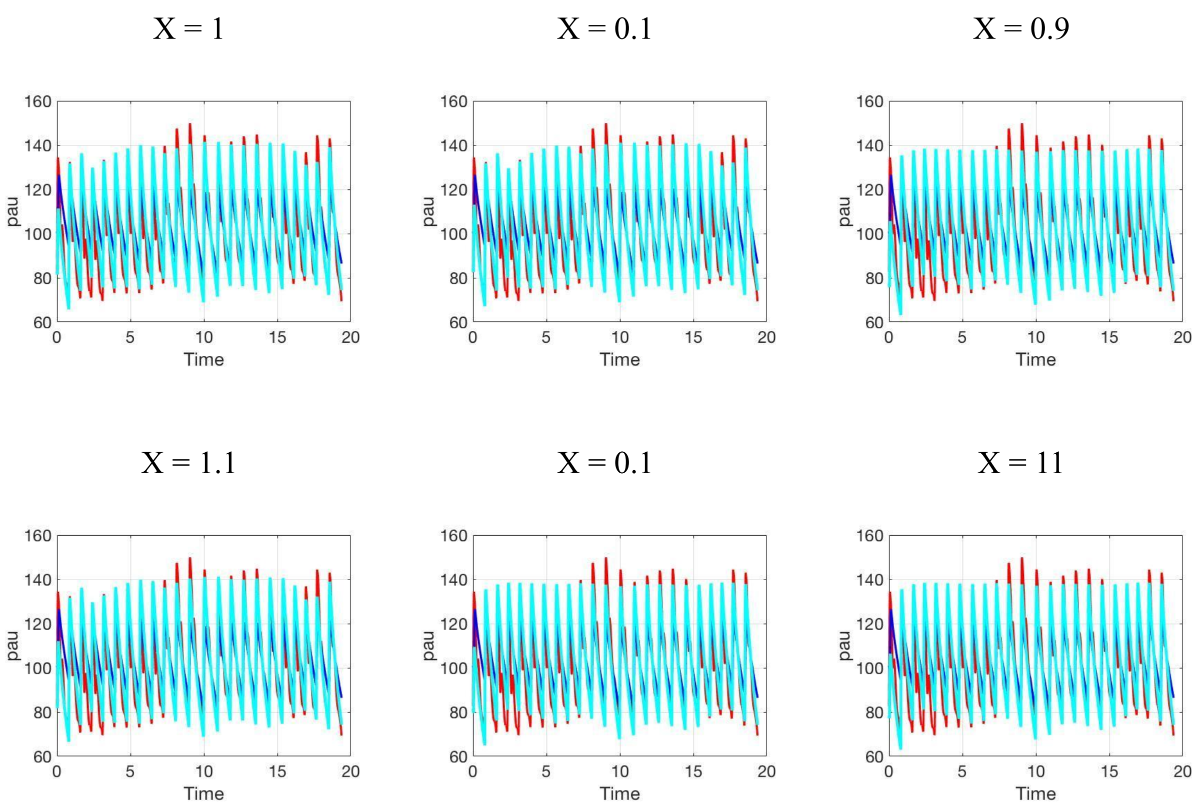


Figure 9. Graphs portray the effects of changing nominal parameters on the dataset.

DISCUSSION

In summary, we used techniques of solving ordinary differential equations, paired with our basic knowledge of human cardiovascular physiology to translate the workings of the heart into a mathematical model. We were successful in finding a more specific understanding of the functions of each subsection of arteries and veins. This was done by varying the compliances, volumes, flows, pressures, and resistances of each subsection and comparing the materialized differences.

Modeling the pressures, volumes, and flows as we changed the parameters allowed us to draw conclusions about how the cardiovascular system works. Our data confirms the relationships given below:

$$C = \frac{\Delta V}{\Delta P} \rightarrow V = CP$$

$$Q = \frac{P_1 - P_2}{R}$$

Examples of these relationships in our model were observed when we halved the parameters individually. When the resistances R_{aup} and R_{alp} were halved, both the pressures P_{au} and P_{al} lowered by around $10\ mmHg$. We believe our basic knowledge of the cardiovascular system can explain this observed change. As the resistance that the blood must move through is lowered, it takes less pressure for the blood to move. A similar effect is seen in the example of a simple circuit in series. If there is a resistor in the circuit, it takes a higher voltage to maintain the same current when compared to a circuit without a resistor ($V = IR$).

Halving the resistances also changed the volume throughout different parts of the body. Halving R_{aup} caused both the diastolic and systolic pressures to decrease, the systolic by $20\ mmHg$ and the diastolic by $40\ mmHg$. As R_{aup} decreases, the blood is more susceptible to the force of gravity, which causes it to pool more during the diastolic phase. Since there is more blood pooling in the lower body, the volume of blood during the systolic phase is lower as well. Halving R_{alp} caused the volume of the blood in the veins to increase, as well as the volume in the upper arteries due to this pooling effect from the lower resistance.

Halving the compliance C_{au} caused the oscillation of the pressure in the arteries to increase, due to more blood being able to quickly flow into and out of the upper body arteries out of the heart. Halving C_{al} caused a similar effect during the diastolic phase of the arterial pressures for the same reason. Halving the venous compliances resulted in similar changes in pressure but lower in magnitude, likely due to the increased distance from the heart compared to the arteries. Lowering C_{vu} caused P_{au} and P_{al} to slightly increase, again due to the increased difficulty for blood to enter the arteries. It also increased the oscillations of P_{vu}, which is possibly reflecting the changes from the arterial pressures. Lowering C_{vl} caused the diastolic component of the arterial pressures to increase. Since blood can more easily move through the veins, the blood will not pool as much, lowering the pressure.

When C_{au} decreases, The upper venous volumes (V_{vu}) increase during both the systolic and diastolic phases. The upper arterial volumes (V_{au}) decrease by around double the magnitude. This implies that more blood is in the veins compared to normal compliance levels. Since the upper arterial compliance is lower, it is harder for the blood to enter the arteries so more blood will pool in the veins. When C_{al} decreases, there is not a noticeable change in the volumes except in V_{al}. It appears that the farther away the changes are from the heart, the lower the effect on the system. Halving C_{vu} and C_{vl} results in similar respective changes as when C_{au} and C_{al} were halved, but the magnitude of the changes was lower.

Elastance is the inverse of compliance, so altering the EM or Em will result in the inverse result of changing the compliances.